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1998 J. Phys.: Condens. Matter 10 4877

(http://iopscience.iop.org/0953-8984/10/22/012)

J. Phys.: Condens. Matter **10** (1998) 4877–4884. Printed in the UK PII: S0953-8984(98)89198-4

# Possible ionic plasmon mediation for the s-wave condensate in optimally doped high-$T_c$ cuprates

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Received 12 November 1997

**Abstract.** Apparently conflicting results of Josephson tunelling experiments in the high-$T_c$ cuprates have been explained by Müller by proposing the coexistence of s-wave and d-wave condensates. We have explained a number of experimental results in the cuprates by assuming that the ionic plasmon promotes the s-wave condensate. As some of the fits are very compelling we propose that the ionic plasmon be accepted as the mediator in the Cooper pairing in this case. We have also explained the existence of a single transition temperature and the existence of the precursor spin-gap phase.

Recent tunelling experiments which were expected to settle the question of Cooper pair symmetry once for all in the high-$T_c$ cuprates (HTCs) gave apparently conflicting results. Tunelling along the $c$-axis of $YBa_2Cu_3O_{6+x}$ (YBCO) clearly showed s-wave character while tunelling along the $ab$-plane indicated ('with one exception') d-wave character of the pair wave function. The conceptual roadblock was removed by Müller [1] by proposing an explanation on the basis of possible coexistence of s-wave and d-wave condensates in the conducting layer. The two condensates are believed to form by pairing, respectively, in the $c$-direction and the $ab$-plane. Elaborating on this picture, we believe that in the doped condition the $Cu^{2+}$ 3d and $O^-$ 2p orbitals would constitute, respectively, the d-wave and s-wave bands. The two bands are not separated in real space but are in the momentum space. They would be almost independent if $t_{pd}$ is small. This is also the condition for strong correlation near half-filling. It has been mentioned that when $t_{pp}$ is not negligible compared to $t_{pd}$ the single-band model fails. Though the experimental evidence favouring the two-band model may not appear extensive at this time [2] we proceed with this study because our main focus, namely, the mechanism promoting the s-wave condensate, is to a degree an independent matter. Our main finding here is that the s-wave condensate is promoted by the *ionic* plasmon (IP) [3].

The experimental evidence for the existence of the IP has not been conclusive [4]. We hope there will be a search for it by transmission electron energy loss spectroscopy. The most important indirect evidence for it has been the result of numerical analysis [5] (hereafter referred to as AMS) of the photoemission data on $Bi_2Sr_2CaCu_2O_{8+x}$ (BSCCO). Assuming s-wave Eliashberg theory AMS obtained for BSCCO $\lambda_s = 8.67$, the transition temperature for IP meditation $T_c = 60$ K, $\Delta_s = 18$ meV and $\mu^* = 0.15$. Apart from the very

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0953-8984/98/224877+08$19.50  

high $\lambda_s$ the other important finding was that a boson of sharp spectral weight distribution at $\omega_{PL} = 10$ meV was the promoter of s-wave superconductivity ($\hbar = k_B = 1$). That such values of $\lambda_s$ and $\omega_{PL}$ become highly plausible if the IP is accepted as the promoter has been suggested before [6] on general theoretical grounds. Here we analyse a number of experimental results on YBCO, the HTC that has been experimentally studied the most, and demonstrate that the assumption of IP ($\omega_{PL} = 10$ meV) explains them quite accurately. In particular, we predict that for optimally doped (OD) YBCO ($x = 0.94$) $\lambda_s = 2.3$ and $T_c = 31$ K; $\Delta_s = 20$ meV is already known [7]. The theory of the IP [3, 8] when adapted to the OD HTC shows that (see below) in BSCCO and YBCO an IP of about 10 meV is expected to be present. Though it would be ideal to identify observables which are specific to the IP and look for them in experiment our nearest approach to this ideal has been the detection of the AMS boson. This is at least one compelling fit. The other evidence we will present here will apply for both the phonon and the IP. The reason why a phonon with the spectral weight of the AMS boson is unacceptable has been mentioned before [6]. The general unsuitability of phonon mediation is discussed widely in the literature.

Since lattice involvement is always indicated in the HTC [9] we have to find a lattice boson. We emphasize that the IP is the only possible lattice boson other than the phonon. It is given by [3, 8]

$$\omega_{PL}(k) = \omega_0 k(k^2 + \chi^2)^{-1/2} \qquad \omega_0^2 = 4\pi n_i e^2/M \qquad \chi^2 = 6\pi n_h e^2 \bar{\eta}/E_F$$
$$\bar{\eta} = [(1-\zeta^2)/4\zeta]\ln|(1+\zeta)/(1-\zeta)| + 1/2 \qquad \zeta = k/2k_F \tag{1}$$

where $\omega_{PL}(k)$ is the IP energy, $M$ is the mass of the O atom where the hole is situated in the doped HTC, $n_i \approx n_h$, $\chi^{-1}$ is the screening length and $\bar{\eta}$ is the Hartree correction factor. The doped HTC has been called an ionic metal because at OD the hole density is one to two orders of magnitude smaller than in ordinary metals. We expect for the s-wave band the usual local density approximation to be a valid band-structure approach (except for $N(E_F)$) particularly at OD. Setting $n_i \approx n_h \approx 3 \times 10^{21}$ units, $E_F = 130$ meV [10] and $m_{eff} = 1.5\, m_e$ [11] one has $k_F = 2.38 \times 10^7$ units (all units are cgs unless mentioned otherwise). So at the Brillouin zone boundary $k_{BZ} = 8.3 \times 10^7$ units, $\bar{\eta} = 2.7 \times 10^{-2}$ and $\chi = 4.1 \times 10^7$ units. Thus from (1) $\omega_0 = 11.3$ meV and $\omega(k_{BZ})_{PL} = 10.1$ meV, in almost exact agreement with AMS. We expect further reduction of $\chi$ due to exchange and correlation so that $k > \chi$ would hold for smaller values of $k$ also. That is, the IP is expected to exist over a large segment of the Brillouin zone. We note that the condition $k > \chi$ provides the escape route from the well known Bohm–Staver mechanism by which the IP would not exist as one would expect $\chi > k$ to be satisfied in ordinary metals. As the figures quoted above show, it is because of the low density of charge carriers in the HTC, $\sim 10^{21}$ units compared to $\sim 10^{23}$ units in ordinary metals, that at least the condition $k_{BZ} > \chi$ is satisfied. Physically, this means that the Coulomb field of a charged ion is not screened enough due to the low density of mobile carriers so that its range extends beyond the nearest neighbour ion. This makes possible the plasma oscillation of the charged ions or the existence of the IP.

Historically, it is the carrier plasmons which were considered as a possible bosonic mediator for Cooper pairing in the HTCs. These theories, however, have not found general acceptance. The single most serious shortcoming of the purely carrier-based theories appears to be that there is no way they can explain the lattice involvement [9] we have mentioned before.

In this work we have focused on the mechanism of pairing in the s-wave band. As already mentioned the two-band model which has been advanced on important experimental grounds in [1] had not been generally accepted earlier. It is now becoming clear, however,

that in the underdoped and overdoped (including OD) regimes the single-band and two-band models, respectively, would be appropriate. The details of this matter are outside the scope of this study. It may be mentioned that the crystals on which the tunelling experiments were done [1] were OD ones, in which $t_{pp}$ is expected to be comparable with $t_{pd}$ and a two-band model appears justified.

An important experiment [12] that indirectly supports the IP mechanism shows that the disappearance of superconductivity in overdoped cuprates is unrelated to the structural phase transition but is determined mainly by the hole concentration in the planes. As can be seen from (1) for high hole concentration the IP would tend to disappear due to the increase of $\chi^2$. The resulting overdoped nonsuperconducting crystal will be in a precursor (spin-gap) phase (see below), a situation now detectable by experiment [13].

To give an explanation of the manifest transition temperature $T_s$ we reproduce briefly, for completeness, the work of [14] and [15], which have discussed in detail the inclusion of Josephson interlayer tunelling into the usual BCS boson exchange theory. In the latter the gap parameter $\Delta(k)$ is defined by $\Delta(k) = -\sum_{k'} V_{k'k} b_{k'}$; this is now modified to $\Delta(k) = T_J b_k - \sum_{k'} V_{k'k} b_{k'}$, where $T_J$ is a general coefficient representing the sum of the Josephson coupling. In [15] this modification of $\Delta(k)$ from the usual BCS value is used to derive the self-consistent equation $b_k = \bar{\chi}(k)[1 - T_J \bar{\chi}(k)]^{-1} \sum_{k'} V_{k'k} b_{k'}$, where $\bar{\chi} = N(E_F) \int_{-\omega_{PL}}^{\omega_{PL}} \mathrm{d}\varepsilon \tanh(2T_s)/\varepsilon$, ignoring the $k$-dependence. By a simple generalization of the BCS theory the preceeding equations lead to $\ln(1.13\omega_{PL}/T_s) = (1 - T_J\bar{\chi})/\lambda_s$, $\lambda_s = N(E_F)V$. It thus appears that $\lambda_s$ is renormalized upwards by a factor $(1 - T_J\bar{\chi})^{-1}$, a result we take over in strong coupling to write

$$T_s = 0.183\omega_{PL}\lambda_s^{1/2}(1 - T_J\bar{\chi})^{-1/2} \qquad \lambda_s > 1.5 \tag{2}$$

where

$$\bar{\chi} = 2N(E_F) \int_0^{\omega_{PL}/2T_s} \mathrm{d}y \tanh y/y \approx N(E_F)\omega_{PL}/T_s$$

since $\omega_{PL}/2T_s \approx 0.58$. For $N(E_F)$ we note the experimental value of 5.8 states/(eV Cu site) for YBCO [16], i.e., 11.6 Cu states/(eV cell). In BSCCO numerical work shows [17] the ratio of Cu states to 0 states to be 0.33/0.30 and, by making the assumption of the same ratio in YBCO, $N(E_F)$ for 0 states is 10.5/(eV cell or pair). We, however, decide on a value of 9.1 for this quantity, the difference being accounted for by the possible unsuitability of the above ratio for YBCO or a fraction of the inner p states being unavailable. We shall further justify this choice later. So with $T_s = 95$ K one has $\bar{\chi} = 11.1$ states/(eV pair) as the pair susceptibility. Also, $T_J = t_\perp^2/t_\parallel$ [14], $t_\parallel = 0.25$ eV and (2) lead to $t_\perp = 0.14$ eV, a magnitude in exact accord with standard band calculation [18].

We notice that for larger $t_\perp$ in YBCO the material is more three dimensional with a smaller $\lambda_s$ but larger $T_J$ while for BSCCO with $t_\perp < 0.1$ eV just the opposite is true. In both the cases because of the presence of two conducting $CuO_2$ layers per unit cell the combined effect of Cooper pairing and Josephson tunelling gives $T_s \approx 100$ K. In $La_{2-x}Sr_xCuO_4$ (LSCO), on the other hand, because of only one conducting layer $T_c \approx 35$ K close to that for YBCO and hardly any Josephson tunelling contribution. In the cuprates with one, two or three $CuO_2$ sheets it is now established that $T_s$ is an increasing function of the number of sheets [19], a fact in keeping with the idea of the Josephson tunelling contribution. With a large number of such sheets the rise of $T_s$ with number should flatten as has been seen experimentally, but there is still no answer to the question of whether there is a maximum in the $T_s$ versus number curve. If there is, the simple picture of Josephson tunelling contribution would need revision.

The value of $N(E_F)$ just assumed leads to the correct value of $\rho_{ab}$ at a high temperature, say, 300 K where the Cu d states are expected to be localized [20] and only the s-wave band is relevant. Writing $\rho_{ab} = 1/n(E_F)e^2\langle v_{ab}^2\rangle_{FS}\,\tau$, where [21] $\tau^{-1} = (4\pi T)\int_0^\infty \mathrm{d}\omega(\alpha^2F(\omega)/\omega)I(\omega/2T)\delta(\omega-\omega_{PL})$ and setting $\lambda_s = 2\alpha^2F(\omega_{PL})/\omega_{PL}$ and noting $I \approx 1$ here,

$$\rho_{ab} = 2\pi T\lambda_s/e^2N(E_F)\langle v_{ab}^2\rangle_{FS}. \tag{3}$$

With $N(E_F) = 9.1$ states/(eV cell) $= 3.28\times10^{34}$ units and $v_F^{ab} = 2.0\times10^7$ units [22] $\rho_{ab} = 1.75\times10^{-4}$ Ω cm. Though earlier work showed higher values of the resistivity possibly due to grain boundary effects [23] later work [24] with a high quality twinned single crystal gave about $1.8\times10^{-4}$ Ω cm. We note that the dominant contribution to the resistivity at higher temperature is the carrier–IP interaction and any other possible contribution is of minor importance.

To explain the small *negative* isotope effect of the planar 0 atom of order $10^{-3}$ [25] which corresponds to an isotope shift $\delta T_s = 0.10$–$0.14$ K one uses (2) and the idea of zero point motion (ZPM) [26] and writes

$$\delta T_s = 0.0915\omega_{PL}\lambda^{1/2}\bar{\chi}(1-T_J\bar{\chi})^{-1.5}[2(t_\perp/t_\parallel)\delta t_\perp - (t_\perp/t_\parallel)^2\delta t_\parallel]. \tag{4}$$

By ignoring $t_{pd}$ compared to $t_{pp}$ and writing $t_\parallel = bt_{pp}^2/E$, $E_{2p}-E_{3d} = 3.6$ eV [27] one has $b = 2.13$ where $t_{pp} = 0.65$ eV has been used. Via the Zener–Slater approximation [28] one writes in confocal elliptic coordinates

$$t_{pp} = A\int \mathrm{d}\upsilon r_A\,\mathrm{e}^{-\kappa(r_A+r_B)} \qquad r_{A,B} = l(\xi\pm\eta) \qquad \mathrm{d}\upsilon = 2\pi l^3(\xi^2-\eta^2)\,\mathrm{d}\xi\,\mathrm{d}\eta$$
$$1\leqslant\xi\leqslant\infty \qquad -1\leqslant\eta\leqslant1$$

which on evaluation gives

$$t_{pp} = Al^2(1+2\kappa l)\,\mathrm{e}^{-2\kappa l} \tag{5}$$

where $\kappa = 1/1.4$ Å is the Slater parameter for the pair of nearest neighbour 0 atoms (taken here approximately as the same as that of the $O_2$ molecule [28]) at a separation $l = 2.78$ Å in YBCO. In (5) $A = 1.44\times10^4$ units by fitting. Also, noting that there would be ZPM of 0 atoms both along $\hat{a}$ and $\hat{b}$ we write

$$\delta t_{pp} = 2Al\,\mathrm{e}^{-2\kappa l}(1+2\kappa l-2\kappa^2l^2)\delta l \qquad \delta t_\parallel = 2bt_{pp}\delta t_{pp}/E \qquad \delta l = \sqrt{2}\delta a$$
$$\delta a = \hat{\kappa}(\Delta u)^2\Delta M/2M \qquad (\Delta u)^2 = \bar{h}/\sqrt{2}M\omega \text{ [29]} \tag{6}$$

where the Slater parameter of the Cu–O bond (of length $a$) $\hat{\kappa} = 1/1.6$ Å and the change of $a$ arising form the substitution $0^{16}\rightarrow0^{18}$ via ZPM of frequency $\omega\approx200$ cm$^{-1}$ [30] $\delta a = 2.74\times10^{-12}$ units; thus $\delta t_\parallel = -1.30\times10^{-16}$ units. We find on evaluation that the first term in square brackets in (4) is negligible in comparison with the second which, on using the above results, lead us to $\delta T_s = 0.103$ K. This is an excellent fit with the experimental value.

In an exhaustive review of a large set of experiments and of HTCs it was concluded that the data could not be explained in terms of carrier concentration alone and the lattice effect needs to be considered [9]. The following is possibly a first calculation of the lattice effect which gives results in quite accurate agreement with experiment. In the same manner as (4) we write for the lattice effect

$$(\mathrm{d}T_s/\mathrm{d}P)_L = -0.0915\omega_{PL}\lambda_s^{1/2}\bar{\chi}(1-T_J\bar{\chi})^{-1.5}[2(t_\perp/t_\parallel)(\mathrm{d}t_\perp/\mathrm{d}P) - (t_\perp/t_\parallel)^2(\mathrm{d}t_\parallel/\mathrm{d}P)]. \tag{7}$$

In the case of YBCO it is expected that pressure along the $b$-direction will affect the chains and cause charge transfer (CT) between the chains and the planes. We assume that pressure

along the $a$-direction will not significantly affect the chains and $\mathrm{d}T_s/\mathrm{d}P_a$ would be almost solely due to the lattice effect. The pressure effect along the $c$-direction on $T_s$ is small and CT in this case has been considered unimportant [31]. In our approximation we have found the overlap between the Cu atom in the plane and the apical O atom to be zero and in any case we expect the lattice effect in the $c$-direction to be negligible. We consider only the uniaxial effects $\mathrm{d}T_s/\mathrm{d}P_i$ $(i = a, b)$ and write from (7)

$$(\mathrm{d}T_s/\mathrm{d}P_i)_L = -0.0915\omega_{PL}\lambda_s^{1/2}\bar{\chi}(1 - T_J\bar{\chi})^{-1.5}(t_\perp/t_\parallel)^2(\mathrm{d}t_\parallel/\mathrm{d}P_i). \tag{8}$$

Using the first two of the equations in (6) $\mathrm{d}t_\parallel/\mathrm{d}l = -3.38 \times 10^{-5}$ units and noting that compression *only* along the $a$-direction gives $\mathrm{d}l = \mathrm{d}a/\sqrt{2}$, $(\mathrm{d}t_\parallel/\mathrm{d}P_a)_L = (\mathrm{d}t_\parallel/\mathrm{d}l)(\mathrm{d}a/\mathrm{d}P_a)/\sqrt{2} = -2.02 \times 10^{-25}$ units. Here we have used $\mathrm{d}a/\mathrm{d}P_a = -8.44 \times 10^{-21}$ units obtained from $c_{11} = 231$ GPa quoted in reference [16] of [30]. So from (8) $(\mathrm{d}T_s/\mathrm{d}P_a)_L = -2.0$ K GPa$^{-1}$ which is in exact agreement with the experimental value of this quantity, $-2.0 \pm 0.2$ K GPa$^{-1}$ [31]. Incidentally, it also confirms our belief that $\mathrm{d}T_s/\mathrm{d}P_a$ is due almost solely to the lattice effect. Similar calculation shows $(\mathrm{d}T_s/\mathrm{d}P_b)_L = -1.75$ K GPa$^{-1}$ using $c_{22} = 268$ GPa [31]. Here for the $b$-direction we have to include the CT effect which is believed to give a positive (negative) contribution to $\mathrm{d}T_s/\mathrm{d}P_b$ for a doping level lower (higher) than OD which is given by $x = 0.94$ for YBCO. Writing $(\mathrm{d}T_s/\mathrm{d}P_b)_{CT} = (\mathrm{d}T_s/\mathrm{d}n_h)(\mathrm{d}n_h/\mathrm{d}P_b)$ and noting that while the first factor on the right is known from (2) the second is calculable via the bond valence sum analysis of structural data [32] one may evaluate $(\mathrm{d}T_s/\mathrm{d}P_b)_{CT}$ for doping levels up to OD. The estimation and evaluation in [33] and [32], respectively, of $\mathrm{d}n_h/\mathrm{d}P_B$ has, however, been widely varying, ranging from $20 \times 10^{-3}$ to $1.7 \times 10^{-3}$ holes/($CuO_2$ formula unit GPa) for the regime $0.4 < x < 1$ and have not been consistent. There is a need for discrimination between the lattice and CT effect which is possible only in a uniaxial experimental and between the regimes $x \gtrless 0.94$. In the present case if we assume a value $\mathrm{d}n_h/\mathrm{d}P_b \approx 17 \times 10^{-3}$ holes/($CuO_2$ formula unit GPa) for $x = 0.9$ we have $\mathrm{d}T_s/\mathrm{d}P_b = (\mathrm{d}T_s/\mathrm{d}P_b)_L + (\mathrm{d}T_s/\mathrm{d}P_b)_{CT} = 1.9$ K GPa$^{-1}$ in agreement with experiment [31]. Figure 4 of [33] shows about one-third of this value of $\mathrm{d}n_h/\mathrm{d}P$ that has been evaluated by using a phenomenological model whose approach and inputs are very different from ours; specifically, it is concerned with uniform compression which cannot separate the lattice and CT effects. Our estimate of $(\mathrm{d}T_s/\mathrm{d}P_b)_{CT}$ can only be vindicated by accurate Hall measurements of $\mathrm{d}n_h/\mathrm{d}P_b$ of well characterized crystals, but our calculation of $(\mathrm{d}T_s/\mathrm{d}P_a)$ is a successful and accurate calculation of lattice effect.

At this point we make some brief comments about the d-wave condensate for the sake of completeness. We believe this condensate can be represented by a one-band $tJ$ model. For such a model it has been shown numerically [34] that there is a BCS-like but $\mathrm{d}(x^2-y^2)$-wave pairing of dressed quasiparticles and we assume antiferromagnetic spin fluctuations of energy $\omega_{SF} \approx 350$ K [35] as the promoter. [34] thus supports the Fermi liquid based approach to the d-wave band and shows that $\Delta_d = 0.15$–$0.27$ $J$ for the electron density $n$ per Cu site varying from 0.5 to 1. Since for the HTC $J = 0.13$ eV for all moderate doping and YBCO has $n = 0.75$ at OD one has $\Delta_d = 27$ meV at OD assuming its linear variation with $n$. Numerical calculation shows [36] that in the d-wave case for $T_d/\omega_{SF} = 0.3$ (1.0) the gap ratio $2\Delta_d/T_d = 6.4$ (4.7). Thus for $T_d = 95$ K $T_d/\omega_{SF} \approx 0.3$ and so the gap ratio and the gap just mentioned returns almost the same value of $T_d$, 97.8 K. This consistency shows the correctness of the approaches [34] and [36] and of our assumption of $\omega_{SF} = 350$ K. It also, incidentally, shows that for OD in YBCO $T_d \approx T_s \approx 95$ K. It can also be seen that for underdoping larger values of $T_d$ are expected. From the data on $\Delta_d$ and $T_d/\omega_{SF}$ mentioned above it is simple to see that $\delta\Delta_d \approx 3\delta T_d$ and $\delta\Delta_d \approx 360\ \delta n$ in temperature units that lead

to $\delta T_d \approx 120\ \delta n$ which in turn, for example, means that for a change from $n = 0.75$ (i.e. carrier density $= 3 \times 10^{21}$ units) to $n = 0.85$ (i.e. carrier density $= 1.8 \times 10^{21}$ units) $T_d$ would rise by about 12 K. This is a prediction for YBCO.

The regime between $T_d$ and $T_s$ in the underdoped case may be called the precursor phase, the existence of which and whose identification with the spin gap phase now appears to be accepted. The appearance of the spin gap phase of d-wave symmetry above the transition temperature $T_s$ and its disappearance at OD in BSCCO has now been seen in experiment [13]. One may visualize that in the spin gap regime there is an incipient d-wave gap (suppression of spectral weight) in the crystal populated by fluctuating Cooper pairs with a floating phase of the order parameter but no global phase coherence and no superconductivity. At the lower temperature $T_s$ the s-wave band also condenses into Cooper pairs and one expected the establishment of global phase coherence by microscopic pair tunelling between the two condensates. This explains the single $T_s = T_d$ for the crystal. One also expects that due to reduction of the spectral weight in the spin gap phase below $T_d$ the resistivity will drop as reflected in the empirical relation $\rho_{ab} \sim T^{\nu}$ ($\nu \approx 2.5$) found in [24].

Lastly, we calculate the electromagnetic coherence lengths $\xi_s^c$ and $\xi_d^{ab}$ at $T = 0$ K for YBCO where the subscripts refer to the gap symmetry and the superscripts to the crystal direction. For the s-wave condensate we use the relation $\xi_s^c \approx \hbar \upsilon_F^c / \pi \Delta_s (1 + \lambda_s)$ [37] with $\upsilon_F^c = 0.6 \times 10^7$ units [22] and $\lambda_s = 2.3$ which lead to $\xi_s^c = 2.9$ Å. For the d-wave condensate intermediate coupling is expected [34, 38] and the relevant equation can be modified to account for anisotropic interaction [38, 39]. Using the relation $\xi(0) = l[\lambda_l(0)/\lambda_L(0)]^2$ one has

$$\xi_d^{ab} \approx \hbar \upsilon_F^{ab} / \pi \Delta_d Z(0) \tag{9}$$

where $Z(0) \approx (1 + \lambda_d)[1 - (\Delta_d/\omega_{SF})^2(1 - \lambda_d)/(1 + \lambda_d)]$; equations (8.20), (B25) and (B26) of [37] have been used here. (The symbol $l$ has been used in a sense different from its earlier use.) Due to anisotropic interaction at intermediate coupling the parameters $\Delta_d$ and $\lambda_d$ are modified [38]: $\Delta_d \longrightarrow \Delta_d(\lambda/(1 + \bar{\lambda}))$ and $\lambda_d \rightarrow (1 + \bar{\lambda})/(1 + \lambda)$ where $\lambda = \langle \lambda_d(k, k') \rangle$ and $\bar{\lambda} = \langle \lambda_d(k) \rangle$; the exact nature of the average has been specified in the reference. A simple representation of the anisotropy has been given [39] by writing $\lambda_d(k, k') = \lambda_d(1 + a_k)(1 + a_{k'})$ and $\lambda_d(k) = \lambda_d(1 + a_k)$ which lead to $\lambda = \lambda_d(1 + x)$ and $\bar{\lambda} = \lambda_d$, $x = \langle a_k^2 \rangle \neq 0$, $\langle a_k \rangle = 0$. We thus have

$$Z(0) = 1 + \frac{1 + \lambda_d}{1 + \lambda_d + \lambda_d x} - \left(\frac{\Delta_d}{\omega_{SF}}\right)^2 \frac{\lambda_d^3 x (1 + x)^2}{(1 + \lambda_d)^2}.$$

Inserting this quantity in (9) one could perform a two-parameter, $\lambda_d$ and $x$, fit using the known value of the relevant length. Instead we have found that a choice $\lambda_d = 0.9$ and $x = 0.9$ gives $Z(0) = 1.17$ and $\xi_d^{ab} = 12.5$ Å. The values of the s-wave and d-wave coherent lengths found above are in very good agreement with the accepted values, 2–3 and 12–13 Å, respectively. The coupling constant of the d-wave condensate is intermediate as expected and the anisotropy parameter $x$ is large. The large anisotropy parameter is expected as there are four zero crossings for the $d(x^2 - y^2)$-symmetry gap.

Summing up, we have calculated numerical values of $t_\perp$, $\rho_{ab}$, $\delta T_s$, $(\mathrm{d}T_c/\mathrm{d}T_a)_L$ and $\xi_s^c$ for OD YBCO which are in almost exact accord with experiment. The only assumption for these has been the magnitude of $N(E_F)$ at O sites and $\lambda_s$, which, being consistent with all these quantities, are expected to be correct. Also, the calculation of $(\mathrm{d}T_s/\mathrm{d}P_a)_L$ illustrates a new theoretical approach. The basic assumption of IP mediation is sustained by the compelling numerical fit of its energy with that of the AMS boson as well as the results

of the above calculation. The final vindication would need experimental detection of the IP in the HTC. The other quantities of which we have found numerical values are $dT_s/dP_b$ and $\xi_d^{ab}$, which have required the input of parameters that seem highly reasonable in the context but for which no more accurate justification is available. We have also explained the existence of the precursor (spin gap) phase of d-wave symmetry which is now known experimentally and have given arguments justifying the existence of only one transition temperature.

This is not an exhaustive study of all possible properties of the HTCs but only some of those which are directly dependent upon the expected IP mechanism of the s-wave condensate. As has been mentioned before [1] it is desirable to determine how the different probes differentiate between or integrate over the two different condensates.

## Acknowledgments

The author wishes to acknowledge Professor J N Islam for support and encouragement and Professor A L Fetter for a helpful comment.

*Note added in proof.* We have modified our opinion on the origin and effect of the spin-gap since this paper was written, our final position has been set forth in a later work awaiting publication.

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